DISCOVERY

NEVILLE de MESTRE

Anyone for tennis?

Tennis is a sport in which the mathematics involves an unusual scoring system together with other applications pertinent to the draw for different types of tournaments and the relative ratios of points won and lost. The name of the sport is thought to have originated from the French word 'tenez', which translates roughly as 'to receive (the ball)'. However, there is an alternative possibility connected with the Egyptian city, Tennis, which was noted in the Middle Ages for its linen from which early tennis balls were made.

The early game 'Real (Royal) Tennis' became popular with the French in the 16th century. The game was also referred to as 'jeu de paume' (palm game), which is connected to the French word 'racket' or 'racquet'. There are Real Tennis courts still in some cities in the world. The most famous is in Paris, which was used for much of the 20th century as an art gallery for many well-known Impressionist paintings. In Australia there are a small number of Real Tennis courts in Hobart, Ballarat, Romsey and Melbourne, while the most recent addition was at Macquarie University in Sydney, built in 1997.

Lawn tennis was invented in 1873 by Major C. Winfield, a British Army Officer. It became popular with the Russian nobility and British upper classes in the late 19th century, but now enjoys worldwide popularity. Ask your students to find out the difference between lawn tennis and royal tennis.

The scoring system is unusual. First of all, zero points are referred to as 'love', probably from the French word 'l'oeuf' for the egg. It is thought that the equivalent zero in cricket, namely a duck, is related to this derivation through duck egg, which looks like a big zero. In tennis, there are 4 points per game (15, 30, 40, game), which were originally (15, 30, 45, 60) sous, being the French monetary value for the amount wagered by players on each point. After some years the call '45' was reduced to '40' for brevity, and '60' was changed to 'game'.

Major tennis tournaments are usually organised in the form of a knockout competition. For a competition with 64 players (or some other power of 2) this is relatively easy to arrange. If you ask your class to determine the number of matches to be played to determine an eventual winner of the whole knockout competition, they will usually develop their answer through the series

$$32 + 16 + 8 + 4 + 2 + 1$$

Many will obtain the total (63) by simple addition on their calculator, but some may be encouraged also to check this as the sum of a geometric progression. This can be done in two ways; either starting with the first term as 32 and the common ratio as 0.5, or starting at the other end with the first term as 1 and the common ratio as 2.

Some students may see that the final answer is one less than the number of players, because there is a one-to-one correspondence between the number of matches and the number of losers. Once this is established then the number of matches when there are not 2^N players becomes trivial. For example, if there are 99 players then there must be 98 matches. But how many byes would there be in the first round for this situation?

Let the number of byes = b
Let the number of matches
in the first round = m
Then $2m + b = 99$

Now the second round should be of the form 2^N (N integer), and should be the nearest power of 2 below 99. (It is revealing for your students to see what happens if they use other powers of 2.) With the above condition

$$m + b = 64$$

providing us with the second of two simultaneous equations, which can now be solved by various means (algebra, graphs, *Excel*) to yield $m = 35$, $b = 29$.

Sometimes a tennis tournament is arranged so that every player plays against every other player. These round-robin tournaments require a draw based on one given in a previous *Discovery* article (de Mestre, 1997a).

In association with developing a draw for such a competition, your students could be asked to calculate the number of matches needed. For example, suppose that 10 members of a tennis club have to play each

other once so that the club champion can be determined. The number of matches needed is 9 for each player, giving a total of 90, which has to be divided by 2 because of duplication (when A plays B, this is the same as B playing A). Therefore, the total number of matches is 45, which can also be used to introduce the combinatorial notation ${}^{10}C_2$ or

$$\binom{10}{2}$$

connected with 10 players and 2 per match. An alternative approach is to say that player A has 9 matches, player B then has only 8 left to play (she has played A already), then player C has 7 left, and hence the total is

$$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$$

This can be obtained by simple addition, or related to the sum of 9 terms of an arithmetical progression (first term 9 and common difference -1, or first term 1 and common difference 1). This problem is related to the well-known *handshake problem* for ten people being introduced to each other.

A slightly more difficult problem for students is to give them the number of matches M and ask them to determine the number of players P. Both the normal and inverse problems could be investigated more generally by creating an *Excel* spreadsheet for P, ranging from 1 to 20 say, versus M.

An interesting feature of tennis relates to the minimum number of points needed to win a match. For example, consider a 3-set match. At first, your students could consider a match with no tie-breakers. The score in sets could be

0–6   6–4   6–4

Note that although the winner of the match (MW) has won 2 sets to the loser's 1, the winner has only won 12 games compared to 14 by the match loser (ML). Further, if all games lost by MW were to love, then in the first set the ratio of winner's points to loser's points would be 0/24. For each of the next two sets suppose that the games won by MW are to 30; but those lost by MW are to love as before. Your students can then calculate that MW points/ML points is 24/28 for each set. Hence, for the match MW/ML points are 48/80, which results in MW winning only 37.50% of the total points in winning this match.

If the 3-set match has the score

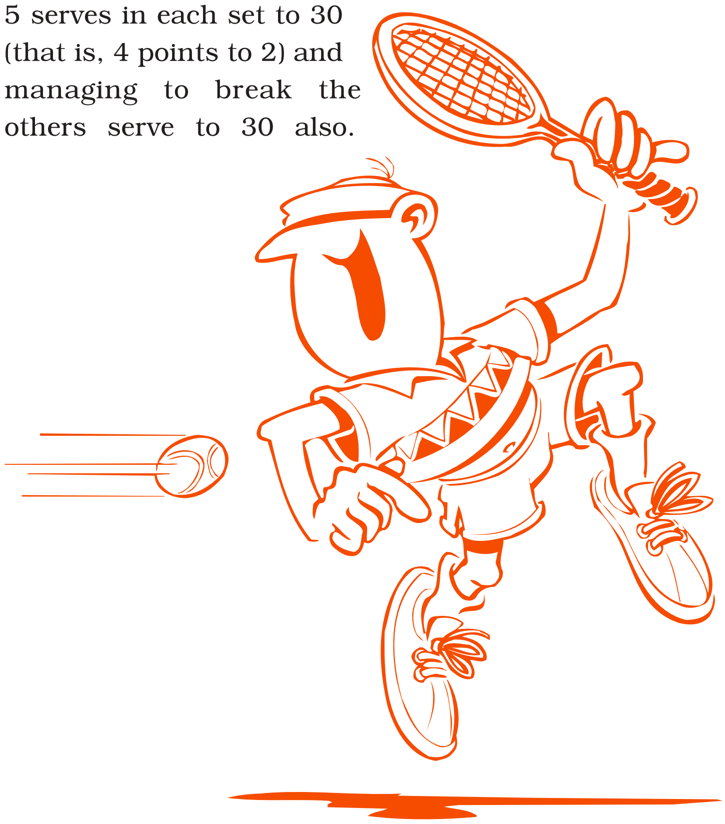
0–6   7–6   7–6

with the score 7–5 in each tiebreaker game in the second and third set, then your students could calculate that for the match, MW/ML points would be 62/106 yielding MW winning only 36.90% of points.

Further calculations can be attempted for 5-set matches. With the score 0–6, 0–6, 6–4, 6–4, 6–4 the ratio MW points/ML points is 72/132 or 35.29% of the points for MW. With the score 0–6, 0–6, 7–6, 7–6, 7–6 the ratio MW/ML = 93/171 or 35.23%, the smallest possible.

Although it seems incredible that a player need only win 36.90% of the points in a 3-set match, or 35.23% in a 5-set match, and still win the match, this situation is highly unlikely, and rarely occurs.

What is more likely is that a player will win a best of 3-set match in 2 sets 6–4, 6–4 losing 4 games in each set to love against a strong server, winning each of his 5 serves in each set to 30 (that is, 4 points to 2) and managing to break the others serve to 30 also.



This calculates as 48 points to the winner and 56 points to the loser or 46.15% of the total points. This, of course, emphasises the strategy in tennis of weaker players saving their energy for the 'big' points.

A problem relating to the tennis court plan can be found in an earlier *Discovery* article (de Mestre, 1997b), while the equipment used can also generate discovery problems through the diminishing bounce of the ball (coefficient of restitution is approximately 0.7) and the area of the racquet (Pick's Theorem or modelled as an ellipse). Real Tennis racquets can also have their area approximated by Pick's Theorem, since they are non-symmetrical. Teachers wanting to explore tennis concepts further can obtain data on the four Grand Slam tennis tournaments can be obtained from the websites listed in the references.

References

de Mestre, N. (1997a). Discovery. *The Australian Mathematics Teacher, 53*(2), 12–13.

de Mestre, N. (1997b). Discovery. *The Australian Mathematics Teacher, 53*(1), 12–13.

http://www.ausopen.com

http://www.frenchopen.org

http://www.usopen.org

http://www.wimbledon.org

ERRATUM

The previous issue of *Discovery* in *AMT 59*(4) included an incorrect diagram of Figure 8 on page 14. The correct diagram is shown below.

Finally here is a dissection and recombination puzzle problem developed by Bolt (1984) which is similar to the tangram puzzle but requires some basic mathematical calculation before your students start to reassemble the pieces. The five pieces for the puzzle are constructed from the 9×6 grid as shown in Figure 8. They have to be recombined to form a square.